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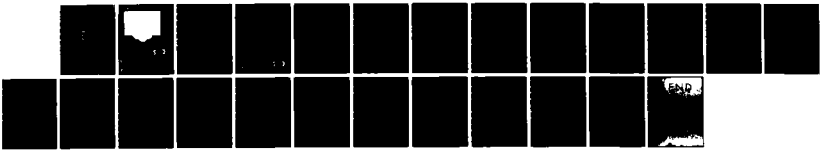

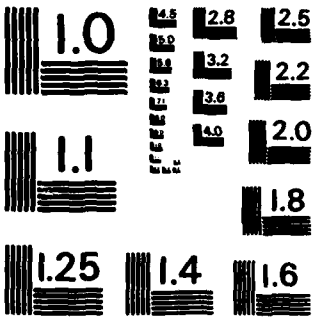

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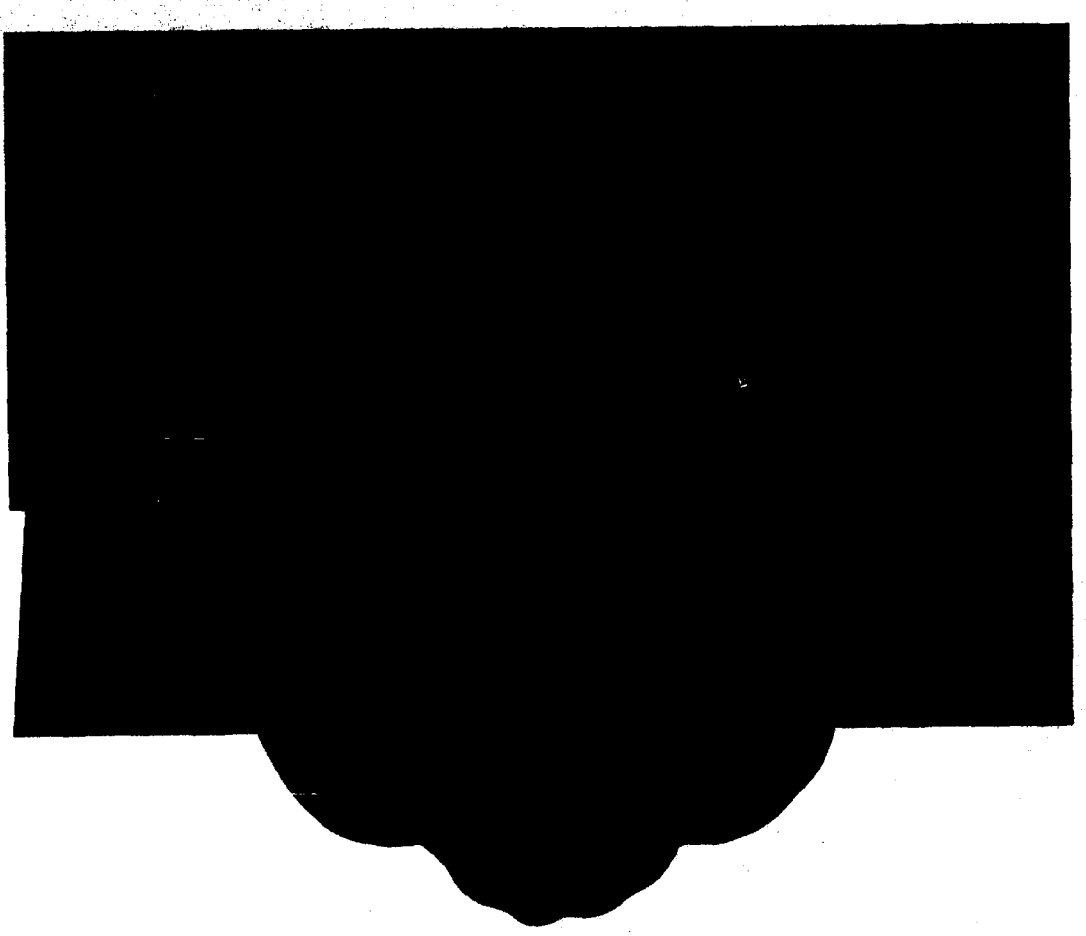

AD-A147 717

THE PERFORMANCE OF VARIABLE SAMPLING PLANS WHEN THE NORMAL DISTRIBUTION IS TRUNCATED

by

Helmut Schneider
University of North Carolina
Chapel Hill, North Carolina

Mimeo Series #1559

1984

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Mimeo Series #1559	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 84-0980
6a. NAME OF PERFORMING ORGANIZATION University of North Carolina 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) Statistics Department, Phillips Hall, 039A, Chapel Hill NC 27514	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F49620-82-C-0009
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2304; TASK NO. A5; WORK UNIT NO.
11. TITLE (Include Security Classification) THE PERFORMANCE OF VARIABLE SAMPLING PLANS WHEN THE NORMAL DISTRIBUTION IS TRUNCATED	
12. PERSONAL AUTHOR(S) Schneider, Helmut	
13a. TYPE OF REPORT Technical 13b. TIME COVERED FROM _____ TO _____	14. DATE OF REPORT (Yr., Mo., Day) JUN 84 15. PAGE COUNT 19
16. SUPPLEMENTARY NOTATION	
17. COSATI CODES FIELD / GROUP / SUB. GR.	18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Robustness; standard variable sampling plans; truncation point.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL MAJ Brian W. Woodruff	22b. TELEPHONE NUMBER (Include Area Code) (202) 767- 5027 22c. OFFICE SYMBOL NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

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SECURITY CLASSIFICATION OF THIS PAGE

84 11 16 019

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INTRODUCTION

Most of the published sampling plans for inspection by variables, e.g. the MIL-STD-414 and BS 6002, assume a normal distribution of the inspected quality characteristic. It is often the case that this assumption is not justifiable, since there are some departures from the normal distribution. Some investigations concern the robustness of the variable sampling plans if the skewness and the excess differ from normality. A summary of work on variable acceptance sampling with emphasis on non-normality is given by Owen (1969). Das and Mitra (1964) use the Cornish-Fisher approximation up to three terms to compute the corresponding probabilities, i.e. those of *rejecting* lots with an acceptable quality level (AQL) and of accepting lots which have limiting quality (LQ). A non-normal distribution with known skewness and excess is assumed. Masuda (1978), using simulation, investigates the robustness of normal sampling plans applied to Student and Lognormal distributions. Schneider and Wilrich (1981) investigate the operating characteristic curve (OC) of a variable sampling plan when the underlying distribution is the Beta-distribution. Furthermore it was pointed out that in the case of a Beta-distribution the Cornish-Fisher approximation is rather poor, since the probabilities of acceptance, computed by this method, are much too high.

Some papers deal with the design of variable sampling plans based on specific distributions such as Gamma (Takagi (1972), Weibull (Hosono et al. (1980)). Srivastava (1961) considers the Cornish-Fisher approximation to design variable sampling plans.

In this paper the effect of truncation is investigated. A truncated normal distribution is suitable in many practical situations where there is a restriction on the variable under consideration. For instance a truncated normal distribution arises in production engineering when sorting procedures eliminate items above or below designated tolerance limits. Truncation often results from technical constraints of a production process. Firstly we will investigate the robustness of standard variable sampling plans with respect to the truncation point. Afterwards we will deal with the design of variable sampling plans based on a truncated normal distribution with given variance σ^2. In the final section the unknown σ case is discussed.

1. Robustness of variable sampling plans

We assume the quality characteristic of the inspected item to be a normally distributed random variable X with parameters μ, σ^2 and truncation point x_r, where σ^2 and x_r are known. A one-sided specification limit U is assigned. Let us consider an upper limit $U \leq x_r$. Items which have $X > U$ are defective. Note that a lower limit L can be treated similarly. The p.d.f. of the quality characteristic X is given as

$$f(x) = \begin{cases} \sigma^{-1}\phi\left(\frac{x-\mu}{\sigma}\right)/\Phi\left(\frac{x_r-\mu}{\sigma}\right) & x \leq x_r \\ 0 & x > x_r \end{cases} \qquad (1)$$

where $\phi(x)$ and $\Phi(x)$ are the standard normal p.d.f. and c.d.f. respectively. The proportion of defective items in a lot is thus

$$p = [\Phi(\frac{x_r-\mu}{\sigma}) - \Phi(\frac{U-\mu}{\sigma})]/\Phi(\frac{x_r-\mu}{\sigma})$$

If we let $\nu_p = (U-\mu)/\sigma$ and $\Delta = (x_r-U)/\sigma$ then we obtain

$$p = 1 - \Phi(\nu_p)/\Phi(\Delta+\nu_p) \qquad (2)$$

According to an agreement between the producer and the consumer, lots with a fraction defective $p \leq AQL$ are presumed to be good and ought to be accepted with probability at least $1-\alpha$. Futhermore lots with $p > LQ$ are not acceptable to the consumer and should be rejected with probability at least $1-\beta$.

Lieberman and Resnikoff (1955) established the following procedure of variables acceptance sampling for the non-truncated case. If the specification is an upper limit, the value t

$$t = \bar{x} + k\sigma \qquad (3)$$

of the test statistic $T = \bar{X} + k\sigma$ is compared with the specification limit U. On the basis of this comparison, each lot is either accepted ($t \leq U$) or rejected ($t > U$). Accordingly, a variables plan is specified by the parameters n (sample size) and k (acceptance constant). The desired (n,k)- plan has to fulfill the condition that the OC-curve of the plan will pass through the points defined by (AQL, $1-\alpha$) and (LQ,β). To compute the sample sixe n and the acceptance constant k, we have to analyse the distribution of T for a given fraction defective p. The OC-curve is given as

$$L(p) = P(T \leq U|p) \qquad (4)$$

If the variance of X is known, the statistic T is normally distributed. Taking into consideration the requirements

$$L(AQL) = P(T \leq U|AQL) = 1-\alpha \tag{5}$$

$$L(LQ) = P(T \leq U|LQ) = \beta \tag{6}$$

it is not difficult to calculate the parameters (n,k). There exist well known published sampling plans such as MIL-STD414 and British STD6002. In order to investigate the robustness of these sampling plans with respect to a truncation, the OC-curve of these plans is examined for the case where X is truncated at x_r. Given a truncation point x_r the expected value and the variance of T are

$$E[T] = \mu - \sigma W(u_r) + k\sigma \tag{7}$$

and

$$V[T] = \frac{\sigma^2}{n}\left[1 - W(u_r)\{W(u_r)+u_r\}\right] \tag{8}$$

respectively, where (see Johnson and Kotz, 1970)

$$W(u_r) = \phi(u_r)/\Phi(u_r) \tag{9}$$

$$u_r = (x_r-\mu)/\sigma = \Delta + \nu_p \, . \tag{10}$$

The OC-curve of an (n,k)-plan is then asymptotically given by

$$L(p) = \Phi(\{\nu_p - k + W(\Delta+\nu_p)\}\sqrt{n\gamma(\Delta+\nu_p)}) \tag{11}$$

where

$$\gamma(\Delta+\nu_p) = [1-W(\Delta+\nu_p)\{W(\Delta+\nu_p)+\Delta+\nu_p\}]^{-1} \qquad (12)$$

and ν_p is the solution to equation (2).

Example

We consider the example AQL = 0.01, LQ = 0.03, α = 0.1 and β = 0.1. In the untruncated case we obtain the sampling plan n = 34, k = 2.106. Applying this plan to a truncated normal distribution the probabilities of acceptance alter according to the amount of truncation. Some probabilities are presented in Table 1.

Table 1

Δ	L(AQL)	L(LQ)
< 0	0	0
0.05	0	0
0.1	0	0
0.2	0.05	0
0.5	0.74	0.01
1.0	0.88	0.07
2.0	0.90	0.094
3.0	0.90	0.095
∞	0.90	0.1

For small values of Δ the probabilities of acceptance differ significantly from $1-\alpha$ and β respectively. This implies for instance that if a producer screens his production near the upper specification limit and

the consumer applies a standard variable sampling plan it is very likely that the lot will be rejected.

In Figures 1 and 2 the OC-curves are plotted for two other examples taken from BS 6002 which is very similar to the MIL-STD-414. Curve I shows the probability of acceptance of an (n,k)-plan if there is no truncation, while curve II shows the probability of acceptance of the same plan after a truncation.

2. The design of a variable sampling plan for known σ

We consider the statistic

$$T' = \hat{\mu} + k\sigma \tag{13}$$

where $\hat{\mu}$ is the maximum likelihood estimate of μ. The likelihood function in the truncated case is

$$L(x_1,\ldots,x_n; x_r,\mu,\sigma) = \exp(-\sum_{1}^{n} (x_i-\mu)^2/2\sigma^2)/[\Phi(u_r)\sqrt{2\pi}\,\sigma]^n \tag{14}$$

Since σ^2 is assumed to be known the maximum likelihood estimate of μ is the solution to

$$\frac{\partial \ln L}{\partial \mu} = -n \frac{\partial \ln \Phi(u_r)}{\partial \mu} + \sum_{i=1}^{n} (x_i-\mu)\sigma^2 = 0 \tag{15}$$

This equation can be rewritten as

$$\hat{\mu} - \bar{x} - \sigma W\left(\frac{x_r-\hat{\mu}}{\sigma}\right) = 0 \tag{16}$$

where $W(x) = \phi(x)/\Phi(x)$. Since (16) cannot be solved analytically, we suggest using one of the following methods.

Figure 1. OC-curve of plan(44,2.17)
(I) untruncated (II) truncated Δ=0.2

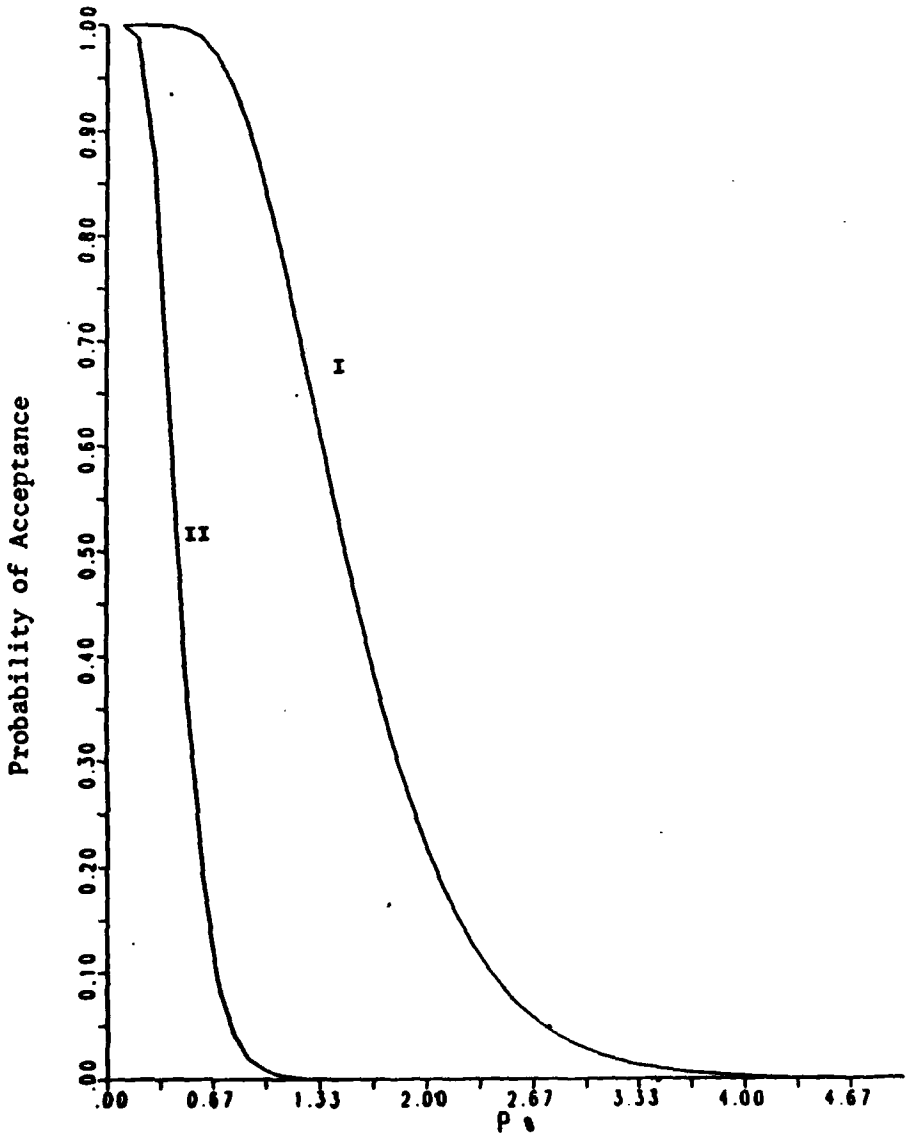
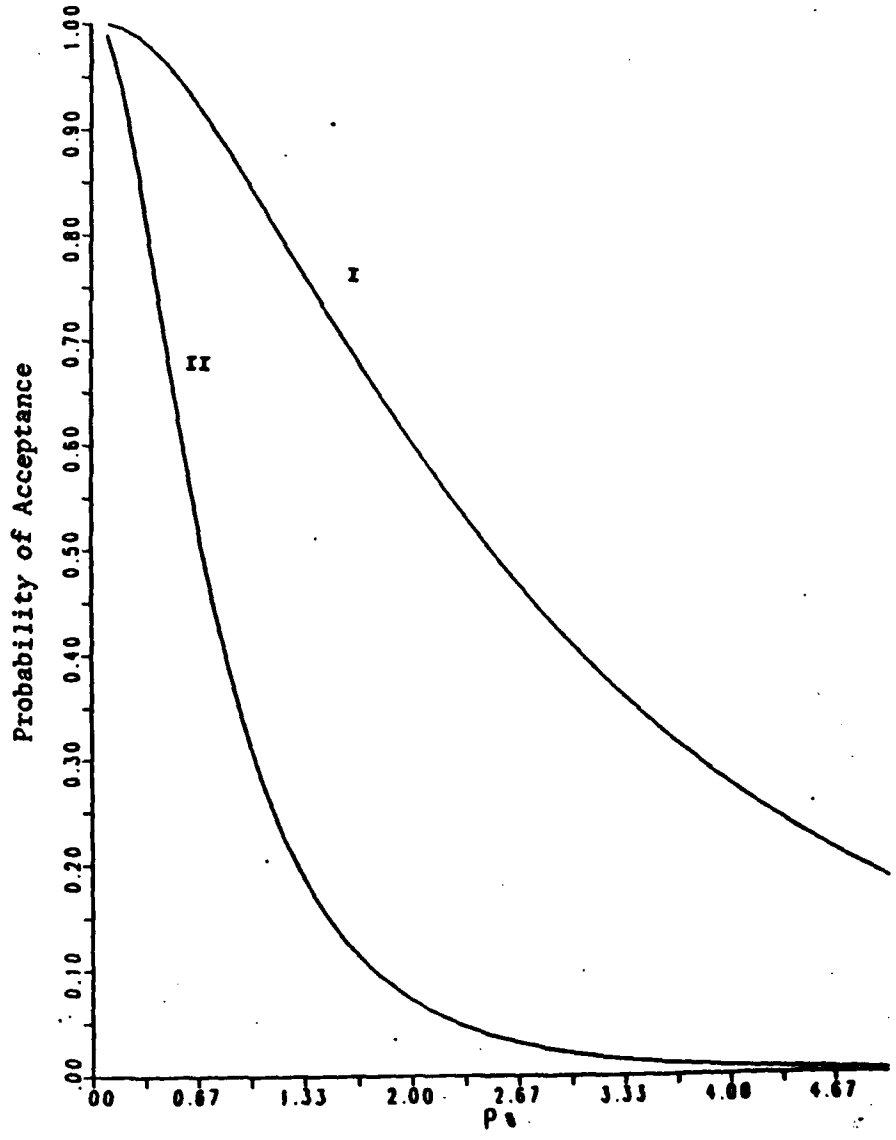


Figure 2. OC-curve of plan (8,1.96)
(I) untruncated (II) truncated Δ=0.2



Taylor Approximation

Let $\theta = (x_r-\bar{x})/\sigma$, then for $\theta \geq 2$ the Taylor approximation might be used.

$$\hat{\mu} = \bar{x} + \sigma W(\theta)/(1+W'(\theta)) \tag{17}$$

where

$$W'(\theta) = -[\phi^2(\theta) + \theta\phi(\theta)\Phi(\theta)]/\Phi^2(\theta) \tag{18}$$

The error is then less than 7% of $(\hat{\mu}-\bar{x})/\sigma$.

Rational Approximation

A common method of obtaining closed solutions for a nonlinear equation with one variable, like (16), is to use rational functions to approximate the functional relationship. If we let $\delta = (\hat{\mu}-\bar{x})/\sigma$ and $\theta = (x_r-\bar{x})/\sigma$, then δ is only a function of θ and the estimates $\hat{\mu}$ are given by

$$\hat{\mu} = \bar{x} + \sigma\delta(\theta)$$

Several rational approximations of the form

$$\delta = P_n(\theta)/P_m(\theta) + \varepsilon(\theta)$$

were tried, where $P_n(\theta)$ and $P_m(\theta)$ are polynoms of degree n and m respectively, and $\varepsilon(\theta)$ is the approximation error. The following approximation for δ is over the range of interest for θ, i.e. it is assumed that the right truncation point is above the mean of the untruncated population.

For left truncation it is assumed that the left truncation point is below the mean of the original population. When less than 50% of the population is truncated then $\theta \geq 0.7979$. If θ lies below 0.7979 the truncation is so severe that a normal distribution should not be used since the variance of $\hat{\mu}$ becomes too large. If θ is larger than 4.33 the truncation will be ignored since the error is less than $10^{-5}\sigma$. For the range where the Taylor approximation does not work we found the rational approximation

$$\delta(\theta) = P_3(\theta)/P_2(\theta) + \varepsilon(\theta) \qquad 0.7979 \leq \theta \leq 2$$

The approximation error is $|\varepsilon(\theta)| \leq 1.3\ 10^{-5}$. When

$$P_3(\theta) = 17.79998379 - 19.306575250 + 7.229392410^2 - 0.935797420^3$$

$$P_2(\theta) = 1 + 12.0234800 2\theta - 3.798744500^2$$

For the range $2 \leq \theta \leq 4.3$ the rational approximation

$$P_3(\theta) = 0.36123448 - 0.2613692 1\theta + 0.0634902 3\theta^2 - 0.0051717 6\theta^3$$

$$P_2(\theta) = 1 - 0.9277505 3\theta + 0.4174792 6\theta^2$$

was derived, where $|\varepsilon(\theta)| \leq 2 \cdot 10^{-5}$. This approximation can be used as an alternative to the Taylor approach, but the Taylor approximation becomes more accurate as θ increases.

Having an estimate of μ the value

$$t' = \hat{\mu} + k\sigma \tag{19}$$

is compared with the upper specification limit U. On the basis of this comparison each lot is either accepted or rejected. In order to determine

a sampling plan such that its OC-curve passes through the points (AQL,1-α) and (LQ,β) the distribution of T' has to be evaluated. It is easily shown that $\hat{\mu}$ is asymptotically normal with mean μ and variance $\frac{\sigma^2}{n}\gamma(u_r)$, where $\gamma(u_r)$ is defined by

$$\gamma(u_r) = [1 - W(u_r)(W(u_r)+u_r)]^{-1} \tag{20}$$

and

$$u_r = \Delta + \nu_p.$$

Hence the OC-curve is approximated by

$$L(p) = p(T' \leq U|p) = \Phi((\nu_p - k)(n/\gamma(u_r))^{\frac{1}{2}}) \tag{21}$$

and subsequently n and k are solutions to

$$L(AQL) = 1-\alpha \tag{22}$$

$$L(LQ) = \beta \tag{23}$$

Let $u_{1-\alpha} = \Phi^{-1}(1-\alpha)$, $u_\beta = \Phi^{-1}(\beta)$ and let ν_{AQL}, ν_{LQ} be the solutions to equation (2) for p = AQL and p = LQ respectively, then we are able to write the equations (22) and (23) in the form

$$(\nu_{AQL} - k)\frac{n^{\frac{1}{2}}}{\gamma(\Delta+\nu_{AQL})^{\frac{1}{2}}} = u_{1-\alpha} \tag{24}$$

and

$$(\nu_{LQ} - k)\frac{n^{\frac{1}{2}}}{\gamma(\Delta+\nu_{LQ})^{\frac{1}{2}}} = u_\beta \tag{25}$$

and thus

$$n^* = \frac{u_{1-\alpha}\gamma(\Delta+\nu_{AQL})^{\frac{1}{2}} - u_{\beta}\gamma(\Delta+\nu_{LQ})^{\frac{1}{2}}}{(\nu_{AQL}-\nu_{LQ})^2}$$

$$k^* = \nu_{AQL} - u_{1-\alpha}\frac{\gamma(\Delta+\nu_{AQL})^{\frac{1}{2}}}{[n^* + 1]}$$

where [x] is the integer part of x.

Tabe 2 shows the sampling plans for the case considered in Example 1.

TABLE 2

Δ	n^*	k^*
0.05	33	-0.79
0.1	18	0.67
0.2	19	1.47
0.5	27	1.96
1.0	32	2.08
2.0	34	2.10
∞	34	2.106

It turns out that the appropriate sampling plan has an even lower sample size than the standard sampling plan. But this is not always the case as some other examples show. We have selected some sampling plans from the British Standard Table III-A single sampling plans for normal inspection master table 'σ' method. We have calculated the true probabilities of acceptance for the case the normal distribution is truncated. Furthermore we have evaluated formulas (26) and (27) to obtain the appropriate

sampling plans which are denoted as n^* and k^*.

Since the evaluation of these sampling plans are based on asymptotic results we have performed a simulation study with 10,000 samples to calculate the probability of acceptance of the (n^*,k^*)-plans for small sample sizes. The results presented in Table 3 show that the suggested plans are able to cope with the situation where a quality characteristic is truncated normally distributed, given that the sample size n is large enough to justify the use of the asymptotic results. When truncation is heavy, i.e. small Δ combined with large p values, the sample size has to be larger than 20 in order to obtain accurate results by using the normal approximation. It can be seen from Table 3 that for small sample sizes (code letter I) and truncation near the specification limit ($\Delta = 0.2$) the probability of accepting lots with p = LQ is significantly smaller than assigned, while the probability of accepting lots with p = AQL is met.

3. The unknown σ case

The maximum likelihood estimates of μ and σ are solutions to the equations

$$\frac{\partial \ln L}{\partial \mu} = 0, \qquad \frac{\partial \ln L}{\partial \sigma} = 0 \tag{28}$$

The solution can be found by the Newton-Raphson method. To avoid iteration we propose the following rational approximation. Firstly, we shall reduce the two equations (28) which have to be solved simultaneously to one equation. To do this we apply the following transformation due to

Table 3.

Simulation results

(n,k) plans taken from BS 6002, $L(p_1)=0.9$, $L(p_2)=0.1$ for the untruncated case.
(n^*,k^*) derived by (26) and (27).
$L(p_1)$, $L(p_2)$, $L^*(p_1)$ and $L^*(p_2)$ are simulation results of 10,000 samples.

AQL	Δ	N	K	p_1	p_2	$L(p_1)$	$L(p_2)$	N^*	K^*	$L^*(p_1)$	$L^*(p_2)$	CODE LETTER
0.25	.20	7	2.25	.0032	.0399	.593	.0	5	1.73	.893	.050	I
	.50					.853	.033	6	2.12	.900	.083	
	1.00					.891	.080	7	2.23	.900	.088	
	2.00					.896	.093	7	2.24	.900	.096	
0.65	.20	8	1.96	.0080	.0646	.476	.0	6	1.34	.890	.046	I
	.50					.839	.028	7	1.81	.893	.079	
	1.00					.891	.085	8	1.94	.899	.089	
	2.00					.898	.105	†9	1.98	.901	.085	
6.50	.20	15	1.13	.0721	.2124	.091	.0	21	.05	.884	.066	I
	.50					.744	.009	13	.79	.890	.076	
	1.00					.883	.060	14	1.05	.900	.089	
	2.00					.899	.097	15	1.13	.904	.098	
0.25	.20	19	2.41	.0035	.0174	.312	.0	12	1.90	.896	.080	L
	.50					.815	.030	17	2.30	.897	.089	
	1.00					.886	.081	19	2.39	.902	.090	
	2.00					.894	.095	†20	2.41	.905	.091	
0.65	.20	23	2.12	.0086	.0324	.139	.0	13	1.48	.900	.089	L
	.50					.777	.016	19	1.97	.900	.091	
	1.00					.883	.074	22	2.09	.912	.098	
	2.00					.896	.094	23	2.11	.906	.097	
6.50	.20	42	1.24	.0759	.1494	.0	.0	46	.13	.891	.081	L
	.50					.544	.002	34	.90	.901	.088	
	1.00					.859	.054	39	1.17	.887	.095	
	2.00					.894	.095	†43	1.24	.901	.083	
0.25	.20	37	2.47	.0037	.0121	.115	.0	22	1.98	.891	.084	N
	.50					.777	.019	31	2.36	.906	.095	
	1.00					.885	.077	36	2.45	.903	.099	
	2.00					.897	.094	37	2.47	.898	.096	
0.65	.20	44	2.17	.0089	.0236	.025	.0	25	1.57	.895	.090	N
	.50					.735	.012	37	2.04	.902	.090	
	1.00					.890	.083	43	2.16	.892	.099	
	2.00					.907	.108	†45	2.18	.902	.094	

† This values deviate from the n values in the tables BS 6002 due to the method of rounding the non integer values n^*.

Cohen (1959).

Let

$$\omega = \frac{s^2}{(x_r-\bar{x})^2} \tag{29}$$

then the equations (28) can be reduced to an equation for u_r .

$$\omega = \frac{1 - W(u_r)(W(u_r)+u_r)}{[W(u_r) + u_r]^2} \tag{30}$$

Let

$$Q(\omega) = \frac{W(u_r)}{W(u_r) + u_r}$$

where u_r is the unique solution to equation 30; then the maximum likelihood estimators can be written as

$$\hat{\sigma}^2 = s^2 + Q(\omega)(x_r-\bar{x})^2 \tag{31}$$

$$\hat{\mu} = \bar{x} + Q(\omega)(x_r-\bar{x}) \tag{32}$$

The following rational approximation for $Q(\omega)$ was derived which can be used when the truncation is less than 50% of the population. Truncation higher than 50% corresponds to $\omega > 0.57081$.

(i) If $\omega < 0.06246$ (corresponds to $u_r > 4$), then set $Q(\omega) = 0$. The maximal absolute error is less than 10^{-5}.

(ii) If $0.06246 \leq \omega \leq 0.57081$ then set

$$Q(\omega) = P_4(\omega)/P_3(\omega) + \varepsilon(\omega)$$

$$P_4(\omega) = -0.00374615 + 0.17462558\omega - 2.87168509\omega^2 + 17.48932655\omega^3 - 11.9171654\omega^4$$

$$P_3(\omega) = 1 + 5.74050101\omega - 13.53427037\omega^2 + 6.88665552\omega^3.$$

The maximal absolute error, is then $|\varepsilon(\omega)| \leq 5.10^{-6}$.

No approximation will be given for $\omega > 0.57081$, since the variances of the estimators become extremely high.

Asymptotic Variance

The variance and covariance of the estimators $\hat{\mu}$ and $\hat{\sigma}$ are obtained by inverting the Fisher information matrix

$$nJ = \begin{bmatrix} -E[\partial^2 \ell nL/\partial^2 \mu] & -E[\partial^2 \ell nL/\partial\mu\partial\sigma] \\ -E[\partial^2 \ell nL/\partial\mu\partial\sigma] & -E[\partial^2 \ell nL/\partial^2 \sigma] \end{bmatrix} = \frac{n}{\sigma^2} \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

where the elements J_{ij} are given by

$$J_{11} = 1 - W(u_r)(W(u_r)+u_r) \tag{33}$$

$$J_{12} = J_{21} = -W(u_r)[1 + u_r(W(u_r)+u_r)] \tag{34}$$

$$J_{22} = 2 + u_r J_{12} \tag{35}$$

The asymptotic covariance matrix (ASCV) is thus

$$\text{ASCV}(\mu_{ML}, \sigma_{ML}) = \frac{\sigma^2}{n} \frac{1}{J_{11}J_{22} - J_{12}^2} \begin{bmatrix} J_{22} & -J_{12} \\ -J_{12} & J_{11} \end{bmatrix} \tag{36}$$

Variable Sampling Plans for Unknown σ

We consider a quality characteristic X which has a right truncated p.d.f as given by (1), in which σ^2 is assumed to be unknown. An upper specification limit $U \leq x_r$ is assigned. Let $\nu_p = (U-\mu)/\sigma$ and $\Delta = (x_r-U)/\sigma$; then we obtain as in Section 1

$$p = 1 - \frac{\Phi(\nu_p)}{\Phi(\Delta+\nu_p)} \tag{37}$$

But now, since σ is unknown, there is no one-to-one relationship between ν_p and p. We will proceed as in the known σ case and define the statistic

$$T = \hat{\mu} + k\hat{\sigma} \tag{38}$$

We know that T is asymptotically normally distributed with expected value

$$E[T] = E[\hat{\mu}] + kE[\hat{\sigma}] \tag{39}$$

and variance

$$V[T] = V[\hat{\mu}] + k^2V[\hat{\sigma}] + 2k\ Cov[\hat{\mu},\hat{\sigma}] \tag{40}$$

where $V[\mu]$, $V[\sigma]$ and $Cov[\mu,\sigma]$ are given by (36).

In principle it is not difficult to find the distribution of T for large n but the operating characteristic curve (OC) will unfortunately depend on the unknown σ. The OC-curve is defined by

$$L(p) = P(T \leq U|p). \tag{41}$$

Thus the OC-curve of an (n,k)-plan is approximately

$$L(p) = \Phi([n / \gamma(\Delta+\nu_p,k)]^{\frac{1}{2}} [(U-\mu)/\sigma-k]) \tag{42}$$

where

$$\gamma(\Delta+\nu_p,k) = \frac{J_{22} + k^2 J_{11} - 2kJ_{12}}{J_{11}J_{22}-J_{12}J_{12}} \tag{43}$$

Since $\gamma(\Delta+\nu_p,k)$ is not only a function of p but also of Δ and thus of σ the OC-curve is dependent on the unknown σ^2.

Table 4

The Probability of Accepting a Lot with AQL and LQ percent defectives for $\sigma' = \sigma, .9\sigma, 1.1\sigma$

σ'	Δ	n	k	AQL	LQ	$L(AQL\|\sigma')$	$L(LQ\|\sigma')$
1.0σ	0.2	20	1.48	.83	4.46	.900	.100
0.9σ						.839	.090
1.1σ						.934	.117
1.0σ	0.5	26	1.95	.810	3.90	.900	.100
0.9σ						.877	.082
1.1σ						.913	.116
1.0σ	1.0	30	2.05	.865	3.97	.900	.100
0.9σ						.893	.094
1.1σ						.905	.104

A rough and ready method is to use the sample variance derived from the lot history, in order to determine a sampling plan for given risks (AQL,1-α) and (LQ,β). Such an approach would be justified if L(p) is not very sensitive to small changes of σ^2, given the sample sizes are sufficiently large. A study of the OC-curve at AQL and LQ for var-

ious Δ and σ shows that the OC-curve can change significantly when the standard deviation differs about 10% from the assumed standard deviation and the trucation is heavy. For instance, when the true standard deviation is 10% lower than assumed, then the probability of accepting a lot at AQL might decrease drastically as shown in Table 4. For example, if $\Delta = 0.2$ then L(AQL) drops from .9 to .839. When the truncation is moderate the differences might be ignored in applications.

In this paper we have considered only the singly truncated normal distribution. The performance of variable sampling plans for a doubly truncated normal distribution can be investigated in a similar way. Unfortunately the estimating procedure becomes more complicated. Only iterative techniques are available.

Acknowledgement

The author's work was supported by the Deutsche Forschungsgemeinschaft and the Air Force Office of Scientific Research and was completed while the author was visiting the Statistics Department of the University of North Carolina at Chapel Hill. The author would like to thank the referees for their valuable suggestions.

Partially supported by the Air Force Office of Scientific Research Grant F49620-82-C-0009.

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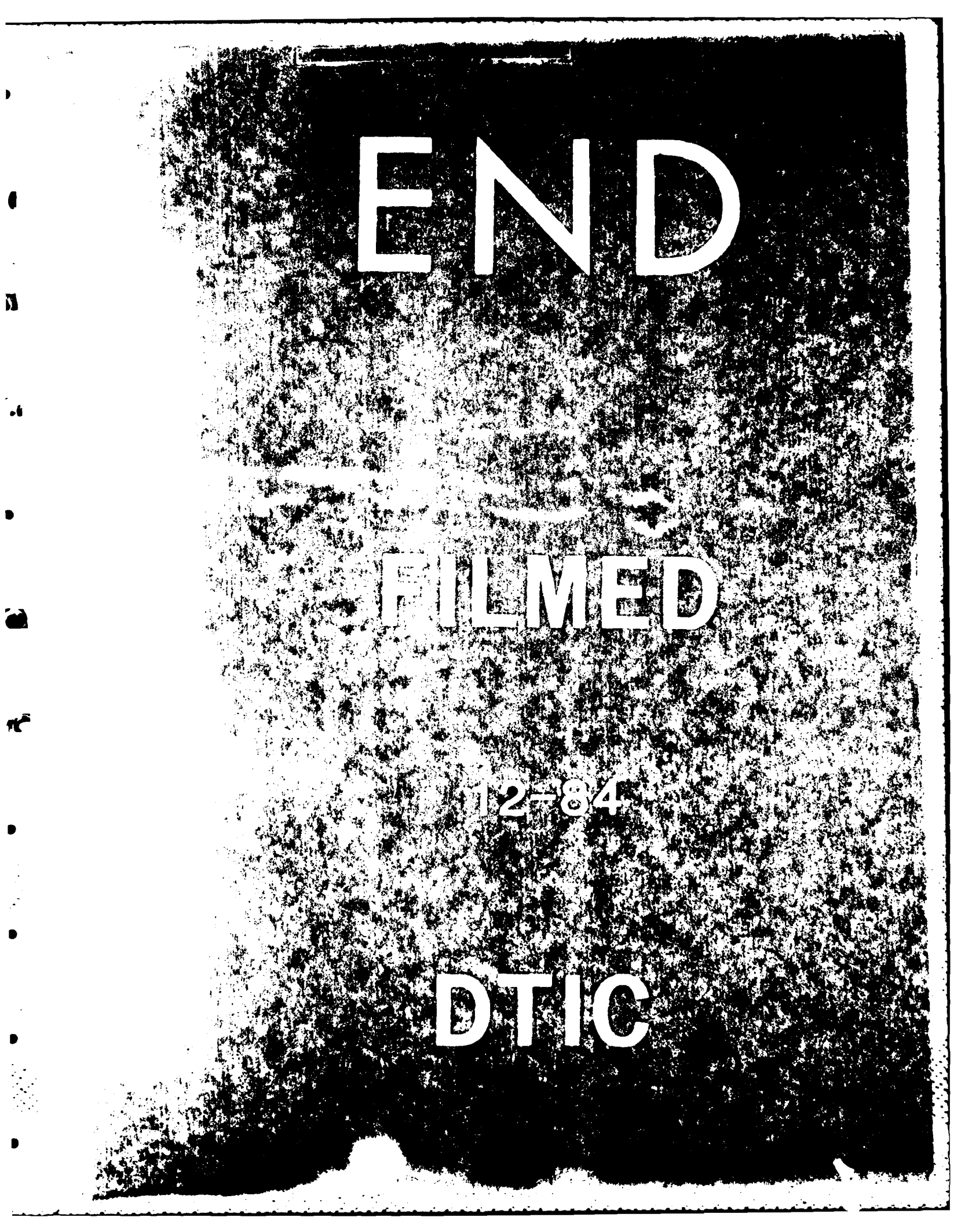
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